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# Wind farm layout optimization in complex terrain: A preliminary study on a Gaussian hill

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**Abstract.** One of the crucial problems for wind farm (WF) development is wind farm layout optimization. It seeks to find the optimal positions of wind turbines (WTs) inside a WF, so as to maximize and/or minimize a single objective or multiple objectives, while satisfying certain constraints. Although this problem for WFs in flat terrain or offshore has been investigated in many studies, it is still a challenging problem for WFs in complex terrain. In this preliminary study, the wind flow conditions of complex terrain without WTs are first obtained from computational fluid dynamics (CFD) simulation, then an adapted Jensen wake model is developed by considering the terrain features and taking the inflow conditions as input. Using this combined method, the wake effects of WF in complex terrain are properly modelled. Besides, a random search (RS) algorithm proposed in previous study is improved by adding some adaptive mechanisms and applied to solve the layout optimization problem of a WF on a Gaussian shape hill. The layout of the WF with a certain number of WTs is optimized to maximize the total power output, which obtained steady improvements over expert guess layouts.

## 1. Introduction

As the main form of large scale wind energy utilization, WFs have been built world widely, both onshore and offshore. Although offshore wind energy is now attracting a lot of interests and has witnessed a rapid increase in recent years, especially in northern Europe, today it represents only about 2% of the global installed capacity of wind energy [1]. For onshore wind energy development, since a lot of suitable sites in flat terrain have already been developed, or due to the mountainous terrain features in certain places, more attention is going to be paid to sites in complex terrain.

Comparing with those built in flat terrain, WFs built in complex terrain benefit from richer wind resource brought by speed-up effects of hills, but they are also exposed to more turbulent flow conditions, higher fatigue loads, more expensive installation, operation and maintenance costs, and other disadvantages [2]. A lot of works in the literature are focused on wind resource assessment in complex terrain. Different models have been developed to simulate wind flows in complex terrain [3], both linear models: mass-consistent models [4], BZ flow model (used in WAsP) [5], and nonlinear models: various CFD methods [6]. Recently, thanks to the fast development of computing capacity, CFD has been widely applied for wind flow simulation in complex terrain because of its accurate flow modeling ability [7-8], it has also been incorporated into the commercial software products, such as WAsP CFD [9].



Optimization of WF layout is a critical task for WF development, which seeks to determine the positions of turbines inside the WF to maximize and/or minimize some objective functions, such as to maximize the energy production and minimize the cost, while meeting various constraints, such as WF boundary, WTs proximity, noise emission level, initial investment limit. In the last two decades, this complex problem has received more and more attentions [10], but most of the works have focused on WFs in flat terrain or offshore WFs, while those in complex terrain have received few investigations. It is mainly due to the difficulty of modelling wake effects. For WFs in flat terrain or offshore WFs, the wake effects can be modeled by engineering wake models, such as Jensen wake model [11-12], with enough accuracy and little computation effort, but when WFs are in complex terrain, these models can't work with satisfactory accuracy. Although CFD method can tackle this difficult problem [13], its computational cost is too high for application in layout optimization, which typically requires thousands of evaluations of different layouts. There are few attempts to utilize other wake models for optimizing WF layout in complex terrain. Song et al. introduced a virtual particle model by simulating wake flow as multiple virtual particles moving convectively and diffusively according to the pre-calculated flow field [15], and applied it in WF layout optimization in complex terrain to maximize the power output, first using a bionic optimization method [16], then a greedy algorithm [17].

In the present paper, a preliminary study on an ideal Gaussian hill is carried out, in which the wind flow field is first obtained from a CFD study [14] and processed to get the speed-up factor map at an interested height level. Then the widely used Jensen wake model is adapted to consider terrain features and complex inflow conditions. Combining these, the power production of a WF with certain layout can be calculated. Also, the previously proposed RS algorithm [18] is advanced by adding some adaptive mechanisms to the search process. Finally, a case study for the WF in a square area on the Gaussian hill is studied, using expert guess layouts based on wind resource consideration, or random layouts as initial layouts. It is shown that the proposed method can obtain steady increase of power production.

## 2. Problem formulation

Since this is a preliminary study which mainly aim to verify the applicability of the proposed adapted wake model and the optimization algorithm, an ideal layout optimization problem in complex terrain is constructed and studied here. Several assumptions are made in the problem formulation, including: an ideal Gaussian shape hill and a square WF area are developed, neglecting the details of real complex terrain, such as local vegetation coverage, tree and forests; only two different wind directions are considered; the effects of turbulence, thermal atmospheric condition are neglected; identical WTs with same hub height are used and the number of WTs are fixed; the power production is determined by the wind speed at hub height combined with the power curve; economic considerations (such as WF cost, electricity selling), design of the WF civil and electrical infrastructure are not included.

### 2.1. Wind flow conditions

To investigate the wind farm layout optimization problem in complex terrain, a model hill is proposed as a Gaussian-shaped ridge, with the terrain shape in  $x$ - $z$  plane defined as:

$$z = H_{hill} \exp\left(-\frac{x^2}{L_{hill}^2} \log 2\right) \quad (1)$$

where  $H_{hill}$  is the height of hill,  $L_{hill}$  is the hill half length at the mid-height of the hill, and  $H_{hill} = L_{hill} = 200m$ . Wind is assumed blowing along the  $x$  direction, and the flow conditions are computed by employing large eddy simulation (LES), using EllipSys3D code with a computational domain of  $[-25H_{hill}, 25H_{hill}] \times [0, 20H_{hill}] \times [-L_{hill}/2, L_{hill}/2]$  in the horizontal, vertical and spanwise directions, respectively. The details of the LES simulation can be found in [14]. In this study, the simulated wind flow conditions are used as an input to the problem. To avoid recirculation zones on the lee side of the hill, the hub height of WTs in this study is assumed to be  $H_{hub} = 100m$  after examining the simulation results, thus the wind flow conditions at this height are of interest here.

Flow field obtained by CFD simulation is typically given as a non-dimensionalized velocity field, which can be used to calculate speed-up factors, wind turnings, turbulence and flow inclination and

assess the wind resource inside the simulated field when combined with the measured wind resource data at a given reference site. Due to the ideal and symmetric shape of the hill, the wake modeling method used and the characteristics of the simulation [14], only the calculated speed-up factors for horizontal wind speed are used in the present study.

In order to utilize the CFD result, the wind flow data at the hub height above the terrain is first extracted, which is along the streamwise direction in the midline of computational zone ( $y = 0$ ). It is noticed that there are some numerical oscillations in the data, especially for the lee side of the hill, which may cause oscillating behaviors of the optimization process. Thus, the extracted data is fitted with a smoothing spline, using Matlab `fit()` function and the speed-up factor of a given position at the hub height level is obtained from the fitted curve, which is denoted as  $S = S(x)$ . Then the inflow wind speed at that position is  $V = V(x) = S(x) \cdot V_0$ , with  $V_0$  represents the reference wind speed, i.e., the inflow wind speed at hub height far away in flat terrain. The flow past the hill in terms of speed-up factors, and the shape of the hill are shown in Fig. 1.

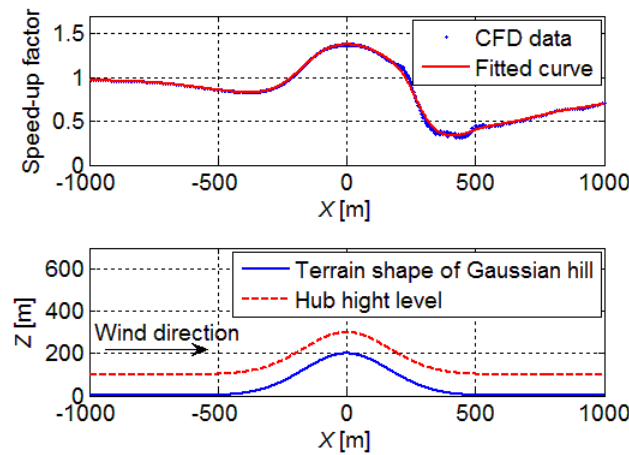


Figure 1: Speed-up factors at the hub height level and shape of the hill

## 2.2. Wake modelling

In this work, Jensen wake model [11-12] is adapted for application in complex terrain, by assuming: (1) centerline of the wake zone behind a rotor follows the surface of terrain along the wind direction; (2) velocity deficit and radius of the wake zone develop linearly according to the traveling distance of the wake; (3) multiple wakes and/or partial wakes merged at each rotor satisfy the kinetic energy deficit balance assumption. It is worthy to point out that the adapting method based on above assumptions is not an innovative contribution, but just a natural generation of the original Jensen wake model to complex terrain. Similar adaptations based on similar assumptions have already been implemented in several commercial software (including WindSim, WindPro, OpenWind, etc.), but the details are usually not presented in published literature. Thus, the detailed wake modelling method for complex terrain is presented in below.

Consider WT $i$  at location  $(x_i, y_i, z_i)$  and WT $j$  at location  $(x_j, y_j, z_j)$  and note that  $z_i$  can be easily obtained as a function of  $x_i$  according to the terrain shape in Eq. (1). Since wind blows along the  $x$  direction, if  $x_i \leq x_j$ , then WT $j$  is at the downwind of the WT $i$  or at the same streamwise level, therefore has no influence on WT $i$ . If  $x_i > x_j$ , then the wake of WT $j$  might affect the rotor of WT $i$  partially, fully or not, depending on the relative locations of the two WTs and the wake development.

Following the similar derivation procedure used in the original Jensen wake model [11-12] and using the assumptions described above, the wind speed and the wake zone radius of the wake of WT $j$  when arriving the streamwise level where WT $i$  located, denoted as  $V_{ij}$  and  $R_{ij}$ , are obtained as:

$$V_{ij} = S(x_j) V_0 \left[ 1 - \frac{1 - \sqrt{1 - C_T (S(x_j) V_0)^2}}{(1 + \alpha (s_{ij}/R_r))^2} \right], \quad (2)$$

$$R_{ij} = \alpha s_{ij} + R_r \quad (3)$$

where  $C_T(\cdot)$  denotes the thrust coefficient of WT at certain wind speeds,  $\alpha$  is the wake decay coefficient,  $R_r = D/2$  represents the radius of rotor and  $s_{ij}$  is the curved distance between the 2 WTs along the wind direction, which is calculated based on locations and terrain data.

On the transversal plane of WT*i*'s location ( $x = x_i$ ), the rotor of WT*i* and the wake zone of WT*j* can be represented as two circles, with radius  $R_r$  and  $R_{ij}$  respectively, located at the same height. Depending on the transversal distance between their centres, which can be denoted as  $d_{ij} = |y_i - y_j|$ , the rotor of WT*i* might be in full wake, in partial wake or out of wake of WT*j*, as shown in Fig. 2.

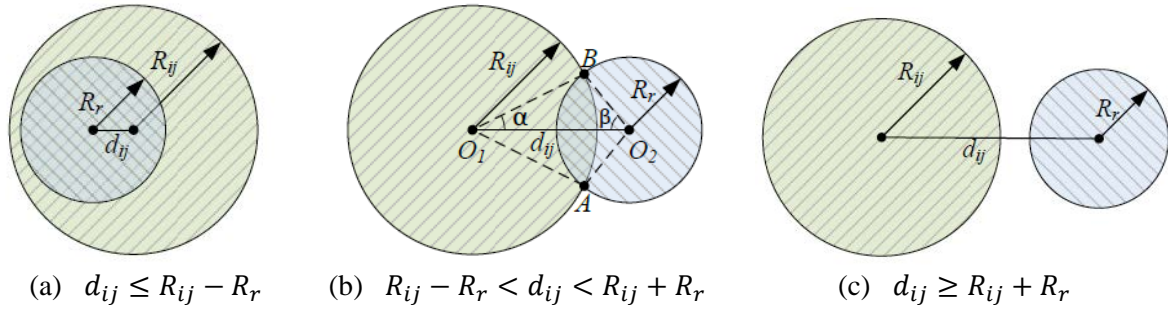


Figure 2: Affected area of WT*i*'s rotor by WT*j*'s wake, shown as the overlapping area of the two circles, in 3 situations: (a) full wake, (b) partial wake, (c) out of wake

It is easy to see that the overlapping area of the two circles is  $A_{ol} = A_r = \pi R_r^2$  in situation (a) and  $A_{ol} = 0$  in situation (c). In situation (b), the calculation requires some basic plane geometry methods.

As shown in Fig. 2. (b), the overlapping area can be derived as the sum of the two circular sector areas ( $O_1\widehat{AB}$  and  $O_2\widehat{AB}$ ) minus the two triangle areas ( $\Delta O_1AO_2$  and  $\Delta O_1BO_2$ ), i.e.,  $A_{ol} = A_{O_1\widehat{AB}} + A_{O_2\widehat{AB}} - A_{\Delta O_1AO_2} - A_{\Delta O_1BO_2}$ . Noticing that the 3 edges of the triangle  $O_1BO_2$  are given as  $O_1B = R_{ij}$ ,  $O_1O_2 = d_{ij}$ ,  $O_2B = R_r$ , we can easily calculate: the two angles  $\alpha = \angle BO_1O_2$  and  $\beta = \angle BO_2O_1$  according to the law of cosine, and the area  $A_{\Delta O_1BO_2}$  using the Heron's formula. As  $\Delta O_1AO_2$  and  $\Delta O_1BO_2$  are congruent triangles, we can derive that  $\angle BO_1A = 2\alpha$ ,  $\angle BO_2A = 2\beta$ ,  $A_{\Delta O_1AO_2} = A_{\Delta O_1BO_2}$ . Based on these derivations, the overlapping area  $A_{ol}$  in Fig. 2. can be written as

$$A_{ol} = \begin{cases} \pi R_r^2, & d_{ij} \leq R_{ij} - R_r \\ 2\alpha R_{ij}^2 + 2\beta R_r^2 - 2A_{\Delta}, & R_{ij} - R_r < d_{ij} < R_{ij} + R_r \\ 0, & d_{ij} \geq R_{ij} + R_r \end{cases} \quad (4)$$

in which

$$\alpha = \cos^{-1}[(R_{ij}^2 + d_{ij}^2 - R_r^2)/(2R_{ij}d_{ij})], \quad \beta = \cos^{-1}[(R_r^2 + d_{ij}^2 - R_{ij}^2)/(2R_r d_{ij})],$$

$$A_{\Delta} = \sqrt{p(p - R_{ij})(p - d_{ij})(p - R_r)}, \quad p = (R_{ij} + d_{ij} + R_r)/2.$$

Having calculated  $A_{ol}$ , we can define  $A_{ij}/A_r$  as an effective percentage for the wake effect of WT*j* on WT*i*, governed by

$$A_{ij}/A_r = \begin{cases} A_{ol}(R_r, R_{ij}, d_{ij})/A_r, & x_i > x_j \\ 0, & x_i \leq x_j \end{cases} \quad (5)$$

which can be used to weight the velocity deficit when considering WT*j*'s effect on WT*i*.

Suppose there are  $N_{wt}$  WTs in the WF and the layout is represented by  $\mathbf{X} = [x_1, x_1, \dots, x_{N_{wt}}]$ ,  $\mathbf{Y} = [y_1, y_2, \dots, y_{N_{wt}}]$ . Based on the kinetic energy deficit balance assumption [12], we have

$$(S(x_i)V_0 - \bar{V}_i)^2 = \sum_{j=1, j \neq i}^{N_{wt}} \left[ \frac{A_{ij}}{A_r} \cdot (S(x_j)V_0 - V_{ij}) \right]^2, \quad (6)$$

Then the effective wind speed that WT*i* experienced is derived as

$$\bar{V}_i = S(x_i)V_0 - \sqrt{\sum_{j=1, j \neq i}^{N_{wt}} \left[ \frac{A_{ij}}{A_r} \cdot (S(x_j)V_0 - V_{ij}) \right]^2} \quad (7)$$

### 2.3. Power production

The WT used in this study is Vestas V80, which has a rated power of 2.0 MW, a rotor diameter of 80 m, a cut-in wind speed of 3 m/s and a cut-out wind speed of 25 m/s. The turbine's characteristic data is extracted from [19], and its power curve and  $C_T$  curve are shown in Fig. 3.

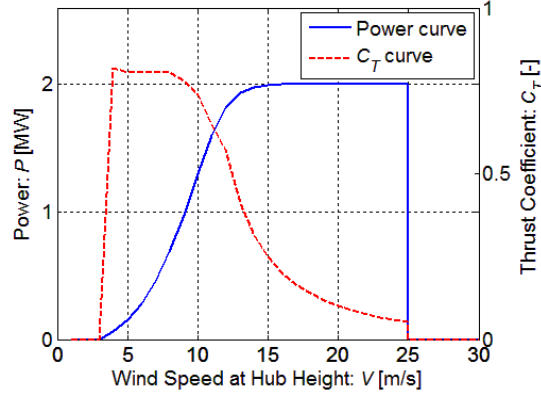


Figure 3: Power curve and  $C_T$  curve of Vestas V80 [19]

Assume the reference wind speed  $V_0$  is described by Weibull distribution, governed by

$$p_{wb}(V_0) = \left(\frac{k}{A}\right) \left(\frac{V_0}{A}\right)^{k-1} \exp\left[-\left(\frac{V_0}{A}\right)^k\right] \quad (8)$$

where  $A$  is the scale factor,  $k$  is the shape factor. As shown in Eqs. (2-7),  $\bar{V}_i$  is a function of reference wind speed  $V_0$  and WF layout  $(\mathbf{X}, \mathbf{Y})$ , using the power curve data, we can get the produced power of WTi as  $P(\bar{V}_i(V_0, \mathbf{X}, \mathbf{Y}))$ , which can be further re-written as  $P_{\bar{V}_i}(V_0, \mathbf{X}, \mathbf{Y})$ , i.e., a function of  $(\mathbf{X}, \mathbf{Y})$  and  $V_0$ . Since we have assumed that  $V_0$  is a random variable with probability density function as Eq. (8), the expected power produced by WTi can be computed as

$$P_i = \int P(\bar{V}_i(V_0, \mathbf{X}, \mathbf{Y})) \cdot p_{wb}(V_0) dV_0 \quad (9)$$

Note that although the Weibull distributions in different sites will be different from that of the reference site due to the terrain effect, it is not necessary to re-calculate them since the inflow wind speed of any given site is determined by the reference inflow wind speed  $V_0$  and the speed up factor at this site  $S(x)$ .

### 2.4. Objective and constraints

The objective of optimization in this study is to maximize the total power produced by WF, by choosing the layout of a WF with certain number of WTs, which satisfies the WF boundary constraint and minimal distance constraints. It is assumed that the shape of WF is a rectangle with  $[X_{min}, X_{max}] \times [Y_{min}, Y_{max}]$ , and the minimal distance between any two WTs is  $Dist_{min}$ . Then the optimization problem can be stated as

$$\max P_{tot} = \sum_{i=1}^{N_{wt}} P_i(\mathbf{X}, \mathbf{Y}) \quad (10)$$

while the layout  $(\mathbf{X}, \mathbf{Y})$  subject to constraints given as follows:

$$X_{min} \leq x_i \leq X_{max}, Y_{min} \leq y_i \leq Y_{max}, \quad \text{for } i = 1, 2, \dots, N_{wt} \quad (11)$$

$$\sqrt{(x_i^2 - x_j^2) + (y_i^2 - y_j^2)} \geq Dist_{min}, \quad \text{for } i, j = 1, 2, \dots, N_{wt} \text{ and } i \neq j \quad (12)$$

Note that in Eq. (12), the distance between two WTs is calculated in a simplified way, not including the vertical differences, which is proper for not so high hill and can save some computation cost.

## 3. RS Algorithm

In the previous study [18], the authors have proposed a RS algorithm for WF layout optimization and applied to an ideal test case in flat terrain, which showed better performance comparing to genetic algorithm (GA). The algorithm starts from an initial feasible layout and then improves the layout

iteratively in the feasible solution space according to the objective and constraints. When moving the WTs during the search process, the old version of RS algorithm selects a WT randomly and moves its position randomly at each 'Random Move' step [18], here this process is improved by remembering and utilizing the information of a good move, i.e., a 'Random Move' step which results in power increase. The procedure of the improved RS algorithm is shown in Algorithm 1.

Algorithm 1: Random search (RS) algorithm for optimization of wind farm layout

---

**Initialize:**

Select initial solution  $S_0$  (from the optimized solution using other algorithms, expert guesses or from a random feasible solution);

Evaluate objective function:  $f_0 = f(S_0)$ ; Set **Improve\_flag** = **.False..**

**While** stop condition is not true:

**1. Random Move**

**If** (**Improve\_flag** == **.False.**):

Select a WT randomly, move its position in a random direction with a random step:

$S = S_0 + \Delta S$ . (Note:  $\Delta S$  is bounded by the long edge of the WF)

**Else:**

Select the WT moved last time, move along the old direction, with a random step size.

**End If**

**2. Feasibility Check**

Check feasibility of  $S$  using constraints of the problem

**If**  $S$  is not feasible:

Repeat the **Random Move** (step 1)

**End If**

**3. Solution Evaluation**

Calculate the objective function of feasible solution  $S$ :  $f = f(S)$

**4. Optimal Solution Update**

**If** ( $f > f_0$ ):

Set  $S_0 = S$ ,  $f_0 = f$ , **Improve\_flag** = **.True..**

**Else:**

Set **Improve\_flag** = **.False..**

**End If**

**End While**

$S_0$  is the optimized solution

---

Note that in Algorithm 1, solution is WF layout  $(X, Y)$ , objective function is  $P_{tot}$ , and feasible solution is the layout that satisfies all constraints in Eqs. (11-12). The stop condition can be set according to different criteria, and in this study, it is chosen to be the maximal evaluation number.

#### 4. Case Study

A WF within a square area with 2000 m by 2000 m, i.e., with  $X_{min} = Y_{min} = -1000$ ,  $X_{max} = Y_{max} = 1000$  in Eq. (11), is considered. The minimal distance between WTs in Eq. (12) is set as  $Dist_{min} = 5D$ . The layout optimization of this WF with 25 WTs and 30 WTs are investigated.

Note that the CFD results used in this study [14] provide only simulation of flow field with wind blowing along the  $x$  direction as shown in Fig. 1, and due to the symmetry of the terrain shape, the flow field with wind blowing along the  $-x$  direction can also be easily obtained. Assuming the reference wind speed  $V_0$  is described by a Weibull distribution as Eq. (8) with  $A = 10.0$ ,  $k = 2.3$ , two wind cases are considered, i.e., one direction wind case (with wind blows always along the  $x$



direction) and two direction wind case (with wind blows along the  $x$  direction in half of the time and the  $-x$  direction in half of the time).

#### 4.1. WF with 25 WTs

Expert guess layout of this WF is constructed based on wind resource consideration and constraints, which is constituted of 5 rows of WTs with 5 WTs in each row and arranged in a staggered grid manner, as shown in Fig. 4.

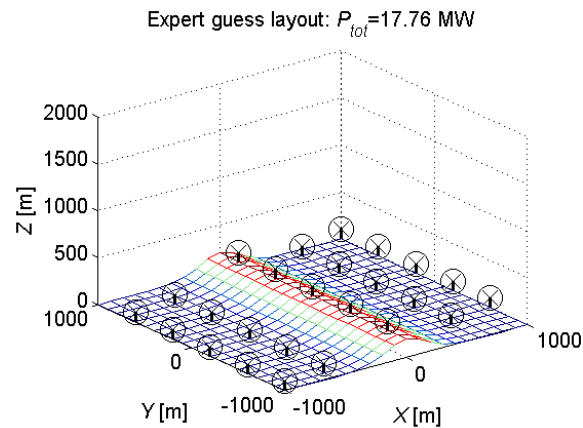


Figure 4: Expert guess layout of a WF with 25 WTs

Using the RS algorithm developed in the previous section, and with the maximal evaluation number set as 100000, the optimized layouts for the two wind cases are shown in Fig. 5. Note that the layouts shown in Fig. 5. are obtained using random layouts as initial layouts, which are better than those obtained using the expert guess layout as initial layouts.

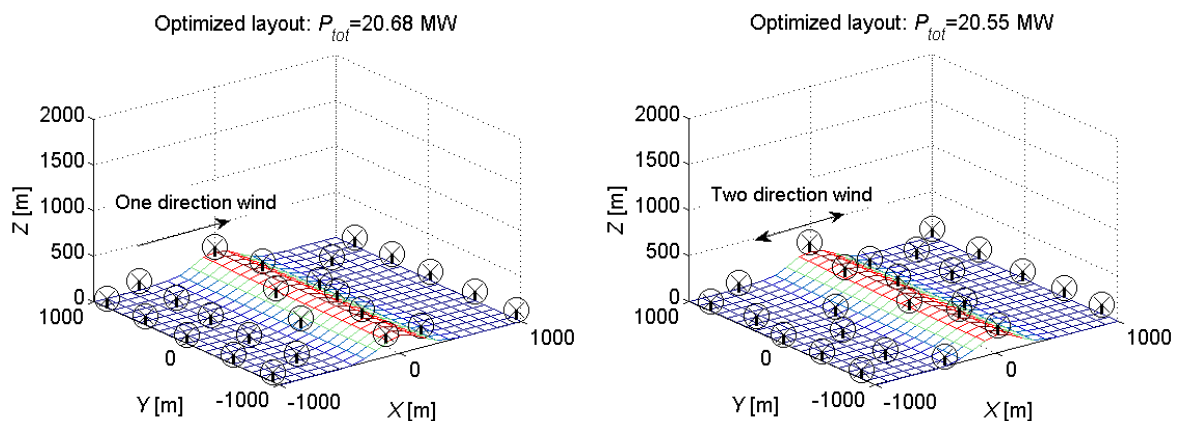


Figure 5: Optimized layouts of the WF with 25 WTs with maximal 100000 evaluations

Due to the symmetry of the original layout and the terrain shape, the power output of the expert guess WF for both wind cases is 17.76 MW. After the optimization, the power output is increased to 20.68 MW for the one direction wind case and 20.55 MW for the two direction wind case.

#### 4.2. WF with 30 WTs

Similarly, the expert guess layout is composed of 5 rows of WTs with 6 WTs in each row and arranged in a grid manner, as shown in Fig. 6.

The optimized layouts for the two wind cases are shown in Fig. 7, with power output of 22.61 MW for the one direction wind case, and 23.09 MW for the two direction wind case, while expert guess layout produces 19.28 MW in both cases.

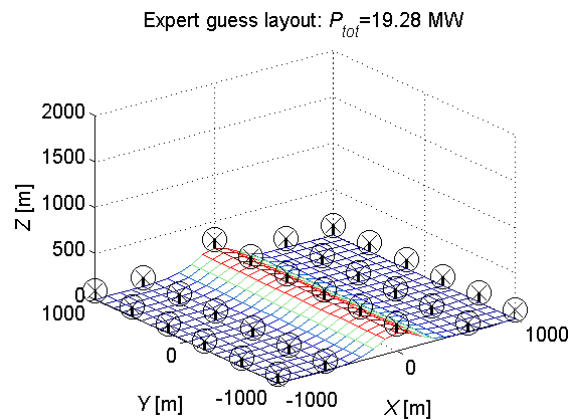


Figure 6: Expert guess layout of a WF with 30 WTs

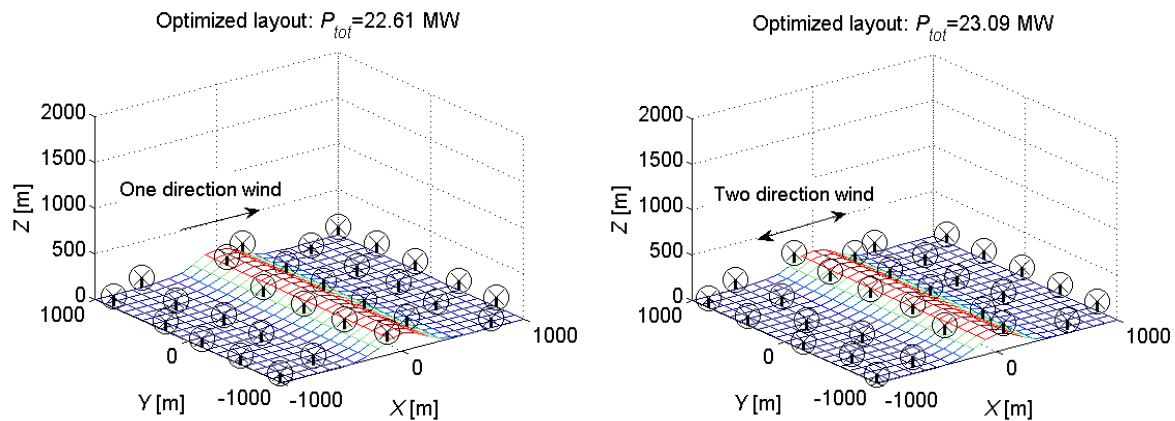


Figure 7: Optimized layouts of the WF with 30 WTs with maximal 100000 evaluations

#### 4.3. Performance analysis

Like other meta-heuristics, the RS algorithm used in this study employs randomness to search for better solutions and stop the process after a certain number of evaluations [20]. Therefore, it usually obtains different results in different runs for the same problem. So it is necessary to run the algorithm for multiple times and study its performance variation.

In this study, the optimization process using RS algorithm is carried out in 10 different runs for each case, either using the expert guess layouts, or using random layouts as initial layouts. The results presented in Figs. 5 and 7 are from the best runs. The details of performance variation for all the cases are shown in Table 1. Both results with a short running time (10000 evaluations) and long running time (100000 evaluations) are presented.

Several characteristics can be observed from the performance variation, including: (1) running the algorithm longer, i.e., with more evaluations, will generally obtain better results, although we can expect this effect will decrease when the algorithm approaching the global optimum after running for long enough time; (2) the robustness of the algorithm is quite good when examining the standard deviation (std) of power in multiple runs, which means we can expect that RS will probably achieve a quite satisfying performance with certain evaluations in all the cases; (3) in multiple runs, the results obtained by using random layouts as initial layouts exhibits larger variance than those using expert guess layouts; (4) the best layout achieved by using random layouts as initial layouts is superior than that achieved by using the expert guess layouts.

Table 1: Performance variation of the RS algorithm in multiple runs

WF	$P_{tot}$ (MW)								
	Expert Guess	Optimized: One direction wind				Optimized: Two direction Wind			
		Best	Worst	Average	Std	Best	Worst	Average	Std
<i>Solution set 1: Using expert guess layouts as initial layouts</i>									
25 WTs	17.76	19.29	18.58	19.09	0.24	18.58	18.51	18.53	0.02
		$N_{eval} = 100000, T_{tot} = 2087.7$ s				$N_{eval} = 100000, T_{tot} = 4220.4$ s			
		19.19	18.41	18.85	0.32	18.54	18.42	18.49	0.03
		$N_{eval} = 10000, T_{tot} = 236.7$ s				$N_{eval} = 10000, T_{tot} = 467.8$ s			
30 WTs	19.28	20.36	20.15	20.25	0.07	20.70	20.33	20.50	0.12
		$N_{eval} = 100000, T_{tot} = 2396.4$ s				$N_{eval} = 100000, T_{tot} = 5059.6$ s			
		20.33	20.03	20.17	0.09	20.66	20.00	20.41	0.20
		$N_{eval} = 10000, T_{tot} = 255.6$ s				$N_{eval} = 10000, T_{tot} = 508.2$ s			
<i>Solution set 2: Using random layouts as initial layouts</i>									
25 WTs	17.76	20.68	18.50	19.65	0.66	20.55	19.15	19.86	0.45
		$N_{eval} = 100000, T_{tot} = 2079.7$ s				$N_{eval} = 100000, T_{tot} = 5950.6$ s			
		20.50	19.20	19.74	0.40	20.52	18.89	20.02	0.46
		$N_{eval} = 10000, T_{tot} = 235.9$ s				$N_{eval} = 10000, T_{tot} = 463.8$ s			
30 WTs	19.28	22.61	21.14	21.71	0.51	23.09	21.60	22.10	0.43
		$N_{eval} = 100000, T_{tot} = 2556.6$ s				$N_{eval} = 100000, T_{tot} = 5126.0$ s			
		21.81	21.02	21.33	0.26	22.48	20.92	21.83	0.50
		$N_{eval} = 10000, T_{tot} = 271.6$ s				$N_{eval} = 10000, T_{tot} = 585.0$ s			
Note: the results presented here is from multiple runs, i.e., for a certain case, the code is running for 10 times. $N_{eval}$ denotes the number of evaluations used per run, $T_{tot}$ represents the total time consumed by 10 runs. And the time is obtained by implementing the algorithm using Fortran 95 on a Dell® laptop with an Intel® i5-2520M CPU @ 2.50GHz.									

## 5. Conclusion

A preliminary study of wind farm layout optimization on a Gaussian hill is presented. Combining a wind flow field from CFD simulation and the adapted Jensen wake model, the authors employ a RS algorithm to maximize the power output of a WF with a certain number of WTs, with consideration of WF boundary constraint and minimal distance constraints between any two WTs. In the case study, the developed method has shown promising performance in different scenarios.

In practice, it is common to use expert guess layouts for WF development, which are based mainly on wind resource considerations and without proper wake modelling. It is shown in this study that layouts obtained using this kind of expert guess methods are far away from optimal ones, which leaves large improving space for WF layout optimization. The RS algorithm used in this study also exhibits several good features, such as: effectiveness of improving initial layouts in different cases, robust performance in multiple runs. In future studies, the layout optimization problem in complex terrain should be investigated using more realistic problem formulation, and more efficient algorithms. Also the adapted Jensen Wake model will need validations and further adjustments.

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